Stochastic Acceleration of Low Energy Electrons in Plasmas with Finite Temperature.

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ABSTRACT

This paper extends our earlier work on the acceleration of low-energy electrons by plasma turbulence to include the effects of finite temperature of the plasma. We consider the resonant interaction of thermal electrons with the whole transverse branch of plasma waves propagating along the magnetic field. We show that our earlier published results for acceleration of low-energy electrons can be applied to the case of finite temperature if a sufficient level of turbulence is present. From comparison of the acceleration rate of the thermal particles with the decay rate of the waves with which they interact, we determine the required energy density of the waves as a fraction of the magnetic energy density, so that a substantial fraction of the background plasma electrons can be accelerated. The dependence of this value on the plasma parameter $\alpha = \omega_{pe}/\Omega_e$ (the ratio of electron plasma frequency to electron gyrofrequency), plasma temperature, and turbulence spectral parameters is quantified. We show that the result is most sensitive to the plasma parameter α. We estimate the required level of total turbulence by calculating the level of turbulence required for the initial acceleration of thermal electrons and that required for further acceleration to higher energies.

1. INTRODUCTION

The importance of the stochastic acceleration of high energy charged particles by turbulent plasma waves is well known. In our previous work (Pryadko and Petrosian, 1997, hereafter refered to as PP1) we investigated the possibility of acceleration of the low energy

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background (thermal) electrons by this process using the well known formalism developed over the years and specifically the formalism described by Schlickeiser (1989) and Dung and Petrosian (1994). We considered interaction of electrons with plasma waves propagating along the magnetic field lines in a cold plasma and presented analytic and numerical results on the acceleration and scattering time scales for different energies and plasma parameters.

The dynamics of the charged particles is determined by the relationships between different plasma time scales which in turn are governed by the properties of the background plasma. In a cold plasma the most important time scales are the acceleration time τ_a, the pitch angle scattering time τ_{sc} and the time $\tau_{tr} \propto L/v$ needed for a particle with velocity v to traverse the plasma region of size L. In cases when the pitch angle scattering time τ_{sc} is much shorter than the traverse time τ_{tr} the pitch angle distribution of the particles will be nearly isotropic. The relative values of the acceleration and scattering times, determined by the ratio of the pitch angle to momentum (or energy) diffusion rates, also has important dynamic consequences. This ratio varies strongly with the plasma parameters and the pitch angle and energy of the electron. The solution of the transport equation when this ratio is high is well known. In our previous paper (PP1) we showed that this ratio becomes less than one at lower energies which indicates that the usual transport equation derived for cosmic rays may not be applicable for a wide range of energies and plasma conditions. A new transport equation was proposed for non-relativistic electrons in high magnetic field, low density plasmas. In PP1 we also showed (numerically and through asymptotic analytic expressions) that when this ratio is smaller than one the acceleration time scale decreases with decreasing energy and could be short; of the order of inverse gyrofrequency times the ratio of turbulent to magnetic field energy densities. The low energy electrons are accelerated by their interaction with short wavelength (high wave number) plasma waves.

In finite temperature plasma such waves will suffer a substantial damping (cyclotron damping). Thus one more time scale have to be taken into consideration - the damping time of the waves. If the damping rate of the waves by the background thermal plasma is much less than the acceleration rate of the test particle, the results obtained for the cold plasma with some small modifications will be still applicable. In the opposite case the acceleration will be limited to the decay time of the wave, so that for a sustained process the wave production must occur at a faster rate than one would consider for a cold plasma.

In this work we investigate the effects due to the finite temperature of plasma. For this purpose we compare the damping rate of the waves needed for acceleration of a low energy electron with the acceleration rate of the same electron. We determine the level of turbulent energy needed for acceleration of a substantial fraction of the background plasma electrons under different plasma conditions. In §2 we reproduce some of the basic results obtained for

the cold plasma case. The modification of these results for finite temperatures is discussed in §3. In §4 we evaluate the fraction of the background electrons which are accelerated by the waves for different plasma conditions. In §5 we compare the levels of turbulence needed for the initial acceleration of thermal electrons to super-thermal energies with that required for further acceleration to relativistic energies. A short summary is given in §6.

2. BASIC EQUATIONS FOR THE COLD PLASMA

In a cold plasma at all energies the acceleration rate of the electrons is determined by the plasma parameters such as the value of the magnetic field, the energy density of the plasma turbulence, the spectral distribution of the waves and most importantly the plasma parameter α, the ratio of plasma frequency ω_p to gyrofrequency of electrons Ω_e:

$$\alpha = \omega_{pe}/\Omega_e = \sqrt{\delta}/\beta_a = 3.2\ (n_e/10^{10}\ \mathrm{cm}^{-3})^{1/2}(B/100\ \mathrm{G})^{-1}. \tag{1}$$

Here β_a is the Alfvén velocity expressed in units of speed of light and $\delta = m_e/m_i$ is the ratio of electron to proton masses.

We are interested in acceleration of low energy electrons which are interacting mainly with cyclotron and electromagnetic branch of the plasma waves. The second interaction is possible only above some critical energy and when $\alpha < 1$. The electrons which do not have enough energy to interact with the electromagnetic waves may still be in resonance with whistler and/or electron-cyclotron modes. In what follows we present the cold plasma relations for non-relativistic electrons.

Following Dung and Petrosian, 1994 and PP1 and assuming a power law distribution of plasma turbulent energy density as a function of wave vector, $\mathcal{E}(k) = (q-1)\mathcal{E}_{tot}\ K_{min}^{q-1}\ K^{-q}$ (for $K \geq K_{min}$ and $q > 1$), it can be shown that the acceleration time of the particle interacting with the plasma waves is:

$$\tau_a = \frac{p^2}{D_{pp}} = \tau_p \gamma^2 \frac{1}{(1-\mu^2)\sum_{j=1}^{N}\left(\frac{\beta_{ph}(k_j)}{\beta}\right)^2 \chi(k_j)}, \tag{2}$$

where

$$\chi(k_j) = \frac{|k_j|^{-q}}{|\beta\mu - \beta_{gr}(k_j)|}, \tag{3}$$

$\beta = v/c$ and the dimensionless wave vector k_j is one of the roots (maximum of four) of the resonant condition:

$$\omega(k_j) - \mu\beta k_j \mp 1/\gamma = 0, \qquad k_j = K_j c/\Omega_e. \tag{4}$$

Here μ is the particle's pitch angle cosine, γ is the Lorentz factor and the coefficient D_{pp} is a momentum diffusion term that appears in the well known Fokker-Planck equation. The upper and lower signs refer to the right(R) and left(L) hand polarized plasma modes. The wave frequency Ω, in units of electron gyrofrequency Ω_e, is determined from the dispersion relation:

$$\frac{k^2}{\omega^2} = 1 - \frac{\alpha^2}{(\omega \mp 1)\omega}, \qquad \omega(k_j) = \Omega(k_j)/\Omega_e, \tag{5}$$

where, because for non-relativistic electron the resonant frequency $\omega \gg \delta$, we have ignored the ion term of the order of δ. The phase and group velocities of these waves (in units of speed of light): $\beta_{ph}(k_j) = \omega_j/k_j$ and $\beta_{gr}(k_j) = d\omega_j/dk_j$, respectively, can be obtained from relations (4) and (5).

The parameter τ_p, which is a typical time scale in the turbulent plasma, is defined as

$$\tau_p^{-1} = \frac{\pi}{2}\Omega_e \left(\frac{\mathcal{E}_{tot}}{B^2/(8\pi)}\right)(q-1)k_{min}^{q-1}. \tag{6}$$

The parameters important for our problem are Ω_e, α, q, k_{min} and the ratio of plasma turbulent density to magnetic energy density:

$$f_{turb}^{tot} = (8\pi\mathcal{E}_{tot}/B^2). \tag{7}$$

Using the above equations it was shown in PP1 that for semi-relativistic electrons the resonant value of the wave vector is $k_{res} \simeq (\alpha^2/\mu\beta)^{1/3}$ and the interacting wave propagates with phase velocity $\beta_{ph} \simeq c/k$ and group velocity $\beta_{gr} \simeq 2\mu\beta c$. In this case the acceleration time of low-energy electrons with velocity β interacting with electron-cyclotron and whistler waves, averaged over μ, can be approximated by the analytic expression

$$\langle\tau_a\rangle = \frac{2p^2}{\int_{-1}^{1} d\mu D_{pp}(\mu)} \simeq \frac{(2+q)(8+q)}{18}\tau_p \alpha^{\frac{2(q+2)}{3}}\beta^{\frac{7-q}{3}}. \tag{8}$$

3. PLASMA WITH FINITE TEMPERATURE.

Extrapolation of equation (8) to zero energies will give zero acceleration time for $q < 7$. However, such low energies are in resonance manly with electron-cyclotron waves with very high values of the wave vector. The cold-plasma dispersion relation allows the vector k to become infinite at resonance, $\Omega \to \Omega_e$. However, in finite temperature plasma this will not be possible because waves with such high values of k are damped quickly via cyclotron damping. Nevertheless, if the damping rate of the wave which accelerate electrons

of velocity β and pitch angle μ is less than the acceleration rate of such an electron, the results obtained for the cold plasma will be still applicable.

In this section we will investigate the range of plasma parameters for which this condition is satisfied. We will be interested in the case when electrons with energies as low as the thermal energy of the background plasma can be accelerated. To describe the thermal property of the plasma we introduce the "thermal velocity" in units of speed of light: $\beta_{th} = (kT/m_e c^2)^{1/2}$.

The dispersion relation for the electron cyclotron wave in plasma with isotropic and finite temperature is

$$\frac{k^2}{\omega^2} = 1 - \frac{\alpha^2 \delta}{\omega} + \frac{\alpha^2}{\omega |k| \beta_{th}} \, Z(\zeta), \quad \zeta = \frac{\omega - 1}{|k|\beta_{th}}, \tag{9}$$

where $Z(\zeta)$, the plasma dispersion function (see e.g. Stix, 1992), can be expressed in terms of the complex error function:

$$Z(\zeta) = i\sqrt{\pi} e^{-\zeta^2}(1 + Erf(i\zeta)). \tag{10}$$

The second term on the right hand side of (9) describes the ion contribution and is negligible for waves with $\omega \simeq 1$. Since electron-wave interaction is described by the resonant condition (4) we can determine the decay rate of the wave in terms of the energy of the interacting electron.

It is well known that for high energy electrons, $\beta \gg \beta_{th}$, and for typical pitch angles, $\mu \neq 0$, the parameter $\zeta \gg 1$ and the imaginary part of the dispersion function is negligible (see e.g. Gershman, 1953 or Stix, 1958). In this case the dispersion relation (9) reduces to the cold plasma dispersion relation (5). Therefore, for electrons with kinetic energies much greater than the thermal energy the results obtained with the assumption of a cold plasma are valid. For particles with velocities close to the thermal velocity we have to compare their acceleration time with the decay time of the wave that they interact with to make sure that there is enough time for the particle to get accelerated before the wave disappears.

The usual technique to calculate the wave decay rate is to separate the frequency ω into real and imaginary parts, $\omega = \omega_r + i\omega_i$, and use it in equation (11).

3.1. Super-thermal non-relativistic electrons

First we consider the case when $|\omega_i| \ll |\omega_r - 1|$, which as we show below is valid for particles with velocities several times the thermal velocities. In this case the imaginary

part of the parameter ζ is negligible so that, to the second order in β, substitution of the resonant condition in equation (9) gives $\zeta = \mu\beta/\beta_{th}$, independent of ω_r, ω_i or k. The dispersion function $Z(\zeta)$ then can be separated into real and imaginary parts:

$$Z(\zeta) = \frac{X+iY}{\zeta}, \tag{11}$$
$$X \equiv -2\zeta S(\zeta), \quad Y \equiv \sqrt{\pi}\,\zeta\, e^{-\zeta^2}, \quad S(\zeta) \equiv e^{-\zeta^2}\int_0^\zeta dz e^{z^2}.$$

In the limit under consideration here both functions X and Y are independent of ω_r and k. Substituting $\zeta = \mu\beta/\beta_{th}$ into the above equations and neglecting the ion contributions (order of $\delta \ll \omega_r$) we obtain:

$$\frac{k^2}{\omega_r^2} = 1 - \frac{\alpha^2}{\omega_r(\omega_r-1)}\left\{X - \frac{\alpha^2 Y^2}{k^2-\alpha^2 X}\right\} \tag{12}$$
$$\omega_i = \frac{\alpha^2 Y}{k^2-\alpha^2 X}\omega_r. \tag{13}$$

Equation (12) describes the modified dispersion relation for the real part of ω and the equation (13) gives the decay rate of the waves. We can solve equation (12) for k and substitute it into equation (13) to obtain the damping rate. We are interested in particles with energies close to the thermal energy, $\beta \sim \beta_{th}$, so that the argument ζ and the functions X and Y are of order of unity and $k^2 \simeq (1-\omega_r)^{-1} \gg 1$. Taking this into consideration the dispersion relation simplifies to

$$\frac{k^2}{\omega_r} \simeq \frac{\alpha^2}{\omega_r - 1}X. \tag{14}$$

The difference between this dispersion relation and the cold plasma dispersion relation (5), in the limit $\omega_r \longrightarrow 1$, is that the term α^2 is replaced by $\alpha^2 X$. Since X is independent of k this fact allows us to use the value of the resonant vector obtained for the cold plasma case, $k_{res}^3 \simeq \alpha^2/(\mu\beta)$, as a solution in equation (9). This is achieved by simply replacing α^2 by $\alpha^2 X$ in equations (12) and (13) which gives

$$k_{res}^3 \simeq \frac{\alpha^2 X}{\mu\beta}, \tag{15}$$

$$\omega_i \simeq \frac{(\mu\alpha\beta)^{\frac{2}{3}} Y}{X^{\frac{2}{3}}}\,\omega_r \tag{16}$$

Using the above equations we can now evaluate the ratio of the imaginary to real part of ζ: $|\omega_i|/|\omega_r - 1| = Y(\zeta)/X(\zeta)$. To be consistent with the assumption stated above this ratio has to be much less than unity. For $\zeta = \mu\beta/\beta_{th} \simeq 0.8$ we get $Y = X$, therefor the above

solution is restricted to velocities greater than several times the thermal velocity (depending on the pitch angle).

In order to compare the wave decay time and the acceleration time we will average the decay rate ω_i over the pitch angle to get an averaged decay time $\tau_{dec} \equiv 1/<\omega_i>$. Introducing the ratio of the acceleration time to the decay time of the wave, $R = \tau_a/\tau_{dec}$, and using equations (8) and (16) we obtain an approximate dependence of R on the plasma parameters and temperature;

$$R \simeq \alpha^{2+\frac{2q}{3}} \beta_{th}^{3-\frac{q}{3}} \Omega_e \tau_p. \tag{17}$$

For successful acceleration we need R to be less than one or, equivalently τ_p to be less than some value which will depend on temperature, plasma parameter α and the spectral index q. Since τ_p^{-1} is proportional to the fraction of plasma turbulence $f_{turb}^{tot} = \mathcal{E}_{tot}/(B_0^2/8\pi)$, this requirement sets a lower limit on the amount of turbulent energy that is needed for acceleration of plasma electrons with energies of several times the thermal energy.

3.2. Thermal electrons

In order to obtain a solution for the acceleration time of electrons with $\beta \simeq \beta_{th}$ when the condition $|\omega_i| \ll |\omega_r - 1|$ is not valid we have to use a complex ζ in equation (11):

$$\zeta = \zeta_r + i\zeta_i, \quad \zeta_r = \frac{\omega_r - 1}{k\beta_{th}} = \mu\frac{\beta}{\beta_{th}}, \quad \zeta_i = \frac{\omega_i}{k\beta_{th}}. \tag{18}$$

The separation of real and imaginary parts of the dispersion relation in this case results in two equations which are cubic in k and nonpolinomial in ω_i so that the solution for k and ω_i can only be found numerically. The resultant solutions show that as in the cold plasma electrons with velocities of order of thermal velocity interact with waves of frequency close to the electron gyrofrequency, but the resonant wave vector now is of order of unity as opposed to the $k_{res} \gg 1$ values for the zero temperature case (see Figure 11). Using the new dispersion relations we then calculate the phase and group velocity of the resonant waves. Substituting these in equation (2) gives the acceleration time assuming that the power law $\mathcal{E}(k) \propto k^{-q}$ is still applicable. Equating the acceleration time with the decay time of the waves $(1/\omega_i)$ gives a general expression for the critical values of several parameters.

Clearly, given an initial distribution of the wave vector k (e.g. power law) the damping and acceleration will modify the wave spectrum in time and space. An exact solution of the problem requires treatment of the coupled wave-particle kinetic equation. This is beyond the scope of the present paper and will be considered in future publications. The

assumption of a power law spectrum will suffice for our purpose here. As we shall see below the results turn out to be almost insensitive to the spectrum.

4. ACCELERATED FRACTION OF THE BACKGROUND ELECTRONS

Our goal is to determine the plasma conditions at which the number of the accelerated electrons is a significant fraction (say, greater than 10%) of the background electrons. In particular, we want to determine the required level of turbulence for acceleration of such a fraction. From the condition for efficient acceleration, $\tau_a \omega_i \leq 1$, we can find an upper limit on the characteristic time scale τ_p which in turn gives the minimum required plasma turbulence energy density $f_{turb}(\mu, \beta, \alpha, T, q)$. The relation between f_{turb} and τ_p depends on the spectral distribution of the waves. For a power law spectrum this relation is given by equation (6). Note that there is an additional dependence of f_{turb} on the wave spectrum due to the dependence on the spectrum of the normalized acceleration time τ_a/τ_p. As we will show later this effect is small; f_{turb} decreases slightly with increasing q. The main variation of f_{turb} comes from the value of k_{min} in equation (6). The criterion $\tau_a \omega_i \leq 1$ then translates to $f_{turb}^{min}(k > k_{min}) \propto k_{min}^{1-q}$. The value of k_{min} is unknown but it should be at least as low as the minimum wave number of waves in resonance with electrons in the considered range of energies. In this section we concentrate on the dependence of f_{turb}^{min} on other plasma parameters and assume $k_{min} = 1$. Later we will show how to modify the results using the correct value of k_{min}. So in what follows f_{turb} represents $(8\pi \mathcal{E}_{turb}/B^2) k_{min}^{q-1}$. The total turbulent energy, due to all the branches including those with $k < k_{min}$ will be, of course, larger than the above estimate. We shall return to this in the next section.

Following the above procedure we calculate the variation of f_{turb} with μ for a thermal electron ($\beta = \beta_{th}$) and different values of α and for several values of temperature. The results are shown on Figures 1 and 2. It can be seen that in order to have an acceleration time at least as short as the decay time of the waves the level of the turbulence has to be greater than some threshold value f_{turb}^*, corresponding to a minimum of the curves at some value μ_* of the pitch angle cosine. If the level of turbulence is less than this value then the acceleration process will not be efficient because the interacting waves are decaying too fast. However, because the curves in Figures 1 and 2 are nearly flat over a wide range of μ, a slightest increase in the turbulent level above this threshold value will result in accelerating electrons with a wide range of pitch angles.

The fraction of the electrons which are accelerated more rapidly than the waves decay is proportional to $0.5(\mu_2 - \mu_1)$ where μ_2 and μ_1 are the high and low values of the intersection with the curves on Figures 1 and 2 of a horizontal line corresponding to a given value of the

f_{turb}. The variation of this fraction with f_{turb} is shown in Figure 3 for $\beta = \beta_{th}$ at $T \simeq 1$ keV. As evident this fraction increases rapidly with increasing f_{turb} up to a level about 2.5 times the minimum level, and then changes slowly. Thus a level of the wave turbulence equal to $3f^*_{turb}$ will involve more than half of the electrons in the acceleration process. For an isotropic distribution the vertical axis of Figure 3 is proportional to the fraction of electrons accelerated, and for a Maxwellian distribution the proportionality constant for $\beta = \beta_{th}$ is $(4/\sqrt{\pi}) \int_1^\infty e^{-x^2} x^2 dx$.

It is clear, therefore, that knowing the value of f^*_{turb} will allow us to estimate the effectiveness of the stochastic acceleration. Consequently, on Figures 4 and 5 we depict the dependences of this on the temperature and the plasma parameter α. We see that both the temperature and the plasma parameter affect the acceleration rate significantly. As expected the value of f^*_{turb} goes to zero when either temperature goes to zero (cold plasma case) or when we deal with a low density high magnetic field plasma ($\alpha \to 0$). This threshold also depends on the electron's energy. This dependence is shown on Figure 6 for several values of the plasma parameter and temperature. We also note that electrons with larger values of μ (smaller pitch angles) are accelerated by the slower decaying, longer wavelength waves. This effect is demonstrated on Figures 7, 8, and 9 where we plot the value of the pitch angle cosine μ_*, at the minimum of the curves in Figures 1 and 2, versus α (for different temperatures), versus temperature (for different α) and versus β/β_{th} for different values of α and T. Note that for the majority of cases $\mu_* > 0.75$ which indicates that the pitch angle of the accelerated particles will be somewhat anisotropic being beamed along the field lines. This is especially true at lower energies when the scattering time is much larger than the acceleration time or the wave decay time.

Because of the complicated dependence of the pitch angle range $(\mu_2 - \mu_1)$ on plasma parameters and on the electron velocity β the determination of the exact number of electrons that can be accelerated is not straightforward. We estimate this fraction as follows. On Figure 10 we show a contour plot of constant f_{turb} in the velocity - pitch angle cosine plane. For f_{turb} equal to or grater than the value for each curve the electrons with values of β/β_{th} and μ above this curve will be accelerated. We can then calculate the fraction of accelerated electrons as a function of the turbulence level and other plasma parameters.

Assuming an isotropic Maxwellian distribution of the background electrons the fraction of the accelerated electrons is given by

$$F(f_{turb}) = \frac{4}{\sqrt{\pi}} \int_{r_{min}(f_{turb})}^{\infty} (\mu_2(r, f_{turb}) - \mu_1(r, f_{turb})) r^2 e^{-r^2} dr, \tag{19}$$

where $r = \beta/\beta_{th}$ and μ_1, μ_2 and r_{min} are obtained from the contour plot in Figure 10. This

fraction can be represented as a sum of two parts: $F(f_{turb}) = F_{r<3} + F_{r>3}$. Note that for $r_{min} \leq 2$ the second term is less than 1% of the total value. We evaluate the first term numerically and use the analytical solution (17) found in §3 to estimate the second term.

Since the main contribution to the fraction of the accelerated electrons comes from the thermal electrons with $\beta \leq 3\beta_{th}$ we can estimate the minimum wave number k_{min} of waves in resonance with these electrons. This will allow us to determine the required turbulence $f_{turb}(k > k_{min})$ corrected for the k_{min}^{q-1} term discussed earlier. On Figure 11 we show the dependence of the resonant wave vector k_{res} on the cosine of electron's pitch-angle for $\beta = \beta_{th}$ and $\beta = 3\beta_{th}$ and different values of α and T. From this plot we see that in most cases the value of the resonant wave vector is greater than or about unity. Note that $k_{res}(\mu)$ decreases monotonicly with μ and we use the minimum value at $\mu = 1$ which gives the highest demand on the required amount of turbulence. We show the dependence of k_{min} on plasma parameter for two different temperatures on Figure 12. The value of k_{min} increases with α and decreases with T but remains of order of unity in the considered range of plasma conditions.

On Figure 13 we plot $F(f_{turb}^{tot})$ for different values of α, T and spectral index q and using the values of k_{min} from Figure 12. As can be seen from this plot the fraction of electrons which can be accelerated in a time shorter than it takes for the turbulent waves to decay increases rapidly with increasing of the level of turbulence (as a fraction of the total magnetic energy density). As we noticed earlier, less turbulence is required for acceleration of 10 % of the background electrons with higher value of the spectral index q. As expected from the analytic result obtained in §3, the turbulent level required for the acceleration of some given fraction of particles strongly depends on plasma parameter α and temperature.

5. ESTIMATION OF THE TOTAL LEVEL OF PLASMA TURBULENCE

In the previous section we have considered acceleration of low energy ($\beta \leq 3\beta_{th}$) electrons which represent a majority of the electrons in a Maxwellian distribution. For most purposes we need further acceleration to much higher energies. This process will require presence of more turbulence at lower k values or longer wavelengths. We now compare the level of turbulence required for the initial acceleration, when thermal effects are important, with that required for the acceleration beyond $3\beta_{th}$ when the thermal effects can be ignored and the cold plasma relations obtained in PP1 are applicable. Using the results presented in Figure 13 we calculate the value for the level of turbulence with $k \geq k_{min}^{th}$, needed for acceleration of 10% of thermal electrons. Table 1 summarizes these results.

T, keV	$\alpha = 0.1, q = 5/3$	$\alpha = 1, q = 5/3$	$\alpha = 1, q = 3$
0.1	$3\ 10^{-6}$	$5\ 10^{-4}$	$3\ 10^{-4}$
1.0	10^{-4}	10^{-2}	$6\ 10^{-3}$

Table 1: Turbulent energy density as a fraction of the magnetic energy density needed for acceleration of 10% of the background electrons.

The amount of turbulence required for further acceleration to higher energies depends on several factors. The first one is the required acceleration time scale τ_a. For shorter times we need proportionally higher values of turbulence. The second factor is the electron gyrofrequency $\Omega_e = eB/m_e c$. The third factor is the plasma parameter α. In what follows we assume $\tau_a \Omega_e \simeq 10^{10}$ and 10^{11} for $\alpha = 1$ and $\alpha = 0.1$, respectively. This corresponds to the acceleration within a second in plasmas with density $n = 10^9 \mathrm{cm}^{-3}$ and uniform magnetic fields of 100 and 1000 Gs, respectively. Another factor which affects f_{turb} is the shape of its spectrum. For a power law distribution (as assumed in PP1) this depends on the index q and the value of k_{min}. Using the results from PP1 we show the required values of the turbulence with $k^{rel}_{min} < k < k^{th}_{min} \simeq 1$ for several plasma parameters in Table 2. For each energy we use the correspondent value of k^{rel}_{min}, the minimum wave number of the waves which are in resonance with the relativistic electron.

E, MeV	k_{min}	$\alpha = 0.1, q = 5/3$	$\alpha = 0.1, q = 3$	$\alpha = 1, q = 5/3$	$\alpha = 1, q = 3$
10^4	$5\ 10^{-5}$	$1.5\ 10^{-5}$	$3\ 10^{-5}$	10^{-2}	$1.6\ 10^{-2}$
10^3	$5\ 10^{-4}$	10^{-6}	$3\ 10^{-6}$	10^{-3}	$1.6\ 10^{-3}$
10^2	$5\ 10^{-3}$	$2.5\ 10^{-8}$	$3\ 10^{-8}$	$5\ 10^{-5}$	$1.6\ 10^{-4}$

Table 2: Turbulent energy density as a fraction of the magnetic energy density needed for acceleration of high-energy electrons.

From comparison of the values in the two tables one can conclude that in order to have a successful acceleration of at least 10% of the background electrons with the subsequent acceleration to GeV energies on the time scale of $\tau_a = (10^{10} - 10^{11})/\Omega_e$, the turbulent energy carried by the waves with $k < k^{th}_{min}$ should be roughly the same or less than the energy due to the turbulent waves with $k > k^{th}_{min}$. This suggests that the turbulent spectrum could flatten at lower k values having a smaller value of spectral index q. Assuming that one half of the turbulent energy is due to the waves with high wave numbers we can give the very conservative estimation of the total level of the plasma turbulence f^{tot}_{turb} by doubling the

numbers given in Table 1. The very steep behavior of the curves on Figure 13 suggests that a small increase in the turbulence level will lead to a significant increase in the fraction of the accelerated background electrons.

6. SUMMARY

This paper extends our earlier work (PP1) on the acceleration of low-energy electrons by plasma turbulence to include the effects of the finite temperature of the plasma. We investigate the possibility of acceleration of the low energy background (thermal) electrons by this process. We use the well known formalism developed over the years and specifically the formalism proposed by Schlickeiser, 1989 and Dung & Petrosian, 1994. As in our previous paper we consider interaction of electrons with plasma waves propagating along the magnetic field lines. In a finite temperature plasma the high wave number, short wavelength waves needed for the acceleration of low energy electrons are damped. The rate of the damping is given by the imaginary part of the resonant wave frequency ω_i. This does not preclude acceleration by waves of thermal particles. Whether or not thermal electrons will be accelerated depends on whether or not the damping rate of the waves, which can accelerate an electron with velocity β and pitch angle μ, is smaller or larger than the acceleration rate of such an electron.

In the first case the conditions for acceleration will be similar to those found for cold plasmas. In the second case acceleration will be possible if turbulence is generated at a rate equal to or faster than the damping rate. This, of course, increases the energy demand for the acceleration process. In this paper we determine the plasma conditions in which the acceleration rate is faster than the damping rate. Since the acceleration rate depends on the level of turbulence present in the plasma this gives us a lower limit on the level of the required turbulence as a function of other plasma parameters.

We determine this level from comparison of the acceleration rate of the particles with the decay rate of the waves they interact with. The dispersion relation for finite temperature plasma is complex so that a simple analytic solution to the problem is not always possible. In the case of electrons for which $\mu\beta/\beta_{th} \gg 1$ we find an asymptotic analytic solution for the ratio of acceleration to decay times, $\tau_a/\tau_{dec} = \tau_a\omega_i$. For electrons with $\beta < 3\beta_{th}$ we find the exact solution numerically. Since τ_a^{-1} is proportional to the fraction of plasma turbulence $f_{tot} = \mathcal{E}_{turb}/(B_0^2/8\pi)$, the condition $\tau_a\omega_i < 1$ sets a lower limit on the amount of turbulent energy that is needed for acceleration of plasma electrons with energies of order of the thermal energy. We investigate the dependence of this minimum value of the turbulence, f_{turb}^{min}, on plasma parameters and electron's pitch angle and velocity. We show

that the required level of turbulence increases with the plasma parameter $\alpha = \omega_{pe}/\Omega_e$ (the ratio of electron plasma frequency to electron gyrofrequency) and the temperature, and it is almost independent of the spectral index q and the pitch angle for a wide range of pitch angles.

Assuming an isotropic Maxwellian distribution of the background electrons we estimate the approximate fraction of these electrons that can be accelerated for a given turbulence level $f_{turb}(k > k^{th}_{min}(\alpha, T))$. We show that the main contribution to this fraction comes from electrons with $\beta \leq 3\beta_{th}$ which allows us to determine the minimum wave number k^{th}_{min} of the turbulent waves in resonance with these electrons. The value of k^{th}_{min} increases with α and decreases with T and is of order of unity. The fraction of accelerated electrons increases with decreasing α and temperature and a small increase in the turbulence level beyond the minimum level can lead to a significant increase in the fraction of the accelerated background electrons.

We compare the level of turbulence required for the initial acceleration of electrons with velocities of order of thermal velocity, when thermal effects are important, with that required for further acceleration to higher energies, when the thermal effects can be ignored. We conclude that in order to have a successful acceleration of at least 10% of the background electrons with the later acceleration of electrons to GeV energies on the time scale of $10^{10}\Omega_e^{-1}$ to $10^{11}\Omega_e^{-1}$, the turbulent energy carried by the waves with $k < k^{th}_{min}$ should be roughly the same or higher than the energy due to the turbulent waves with $k > k^{th}_{min}$. Thus a very conservative estimate of the total level of the plasma turbulence f^{tot}_{turb} is obtained by assuming that the turbulent energy is equally distributed between the waves with high and low wave numbers. This estimated value does not exceed 2% in the worst case scenario.

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REFERENCES

Dung, R. and Petrosian, V. 1994, ApJ, 421, 550

Gershman, B.N., 1953, Zhur. Eksp. I. Theoret. Fiz., 24, 453

Hamilton, R.J. and Petrosian, V. 1992, ApJ, 398, 350

Jaekel, U. and Schlickeiser, R. 1992, J. Phys., 18, 1089

Lerche, I. 1968, Phys. Fluids, 11, 1720

Melrose D.B. 1974, Solar Phys., 37, 353.

Melrose D.B. 1980, Plasma Astrophysics, (Gordon and Breach, New York).

Miller J.A. and Ramaty R., 1987, Solar Phys., 113, 195.

Miller J.A. 1991, ApJ. 376, 342.

Miller J.A. and Ramaty R., 1992, in: Particle Acceleration in Cosmic Plasmas, eds. G.P.Zank and T.K.Gaiser (AIP, New York).

Pryadko, J. and Petrosian, V. 1997, ApJ., 400, ???

Schlickeiser, R. 1989, ApJ, 336, 243

Steinacker, J. and Miller, J.A. 1992, ApJ., 393, 764

Stix, T.H. 1992, Waves in Plasma, (AIP, New York)

Sturrock, P.A. 1994, Plasma Physics, (Cambridge University Press)

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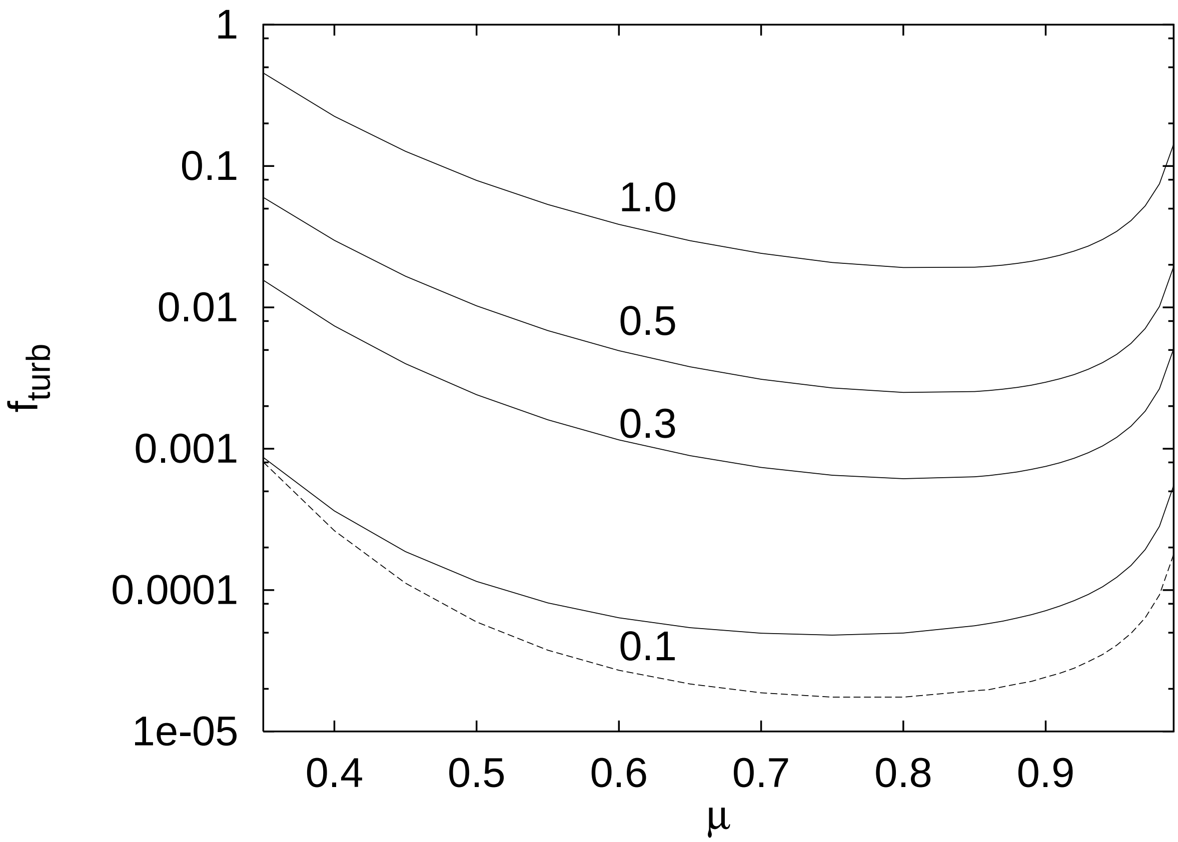

Fig. 1.— The ratio of plasma turbulent energy density, to magnetic energy density $f_{turb} = (8\pi\mathcal{E}_{tot}/B^2)k_{min}^{q-1}$, at which the wave damping rate is equal to the electron acceleration rate, as a function of μ for plasma parameter $\alpha = 1$, 0.5, 0.3 and 0.1 from top to bottom. The plasma temperature is assumed to be 1 keV and $\beta = \beta_{th}$. The spectral index of the wave turbulence is $q = 5/3$ except for the dashed line for which $q = 3$ and $\alpha = 0.1$.

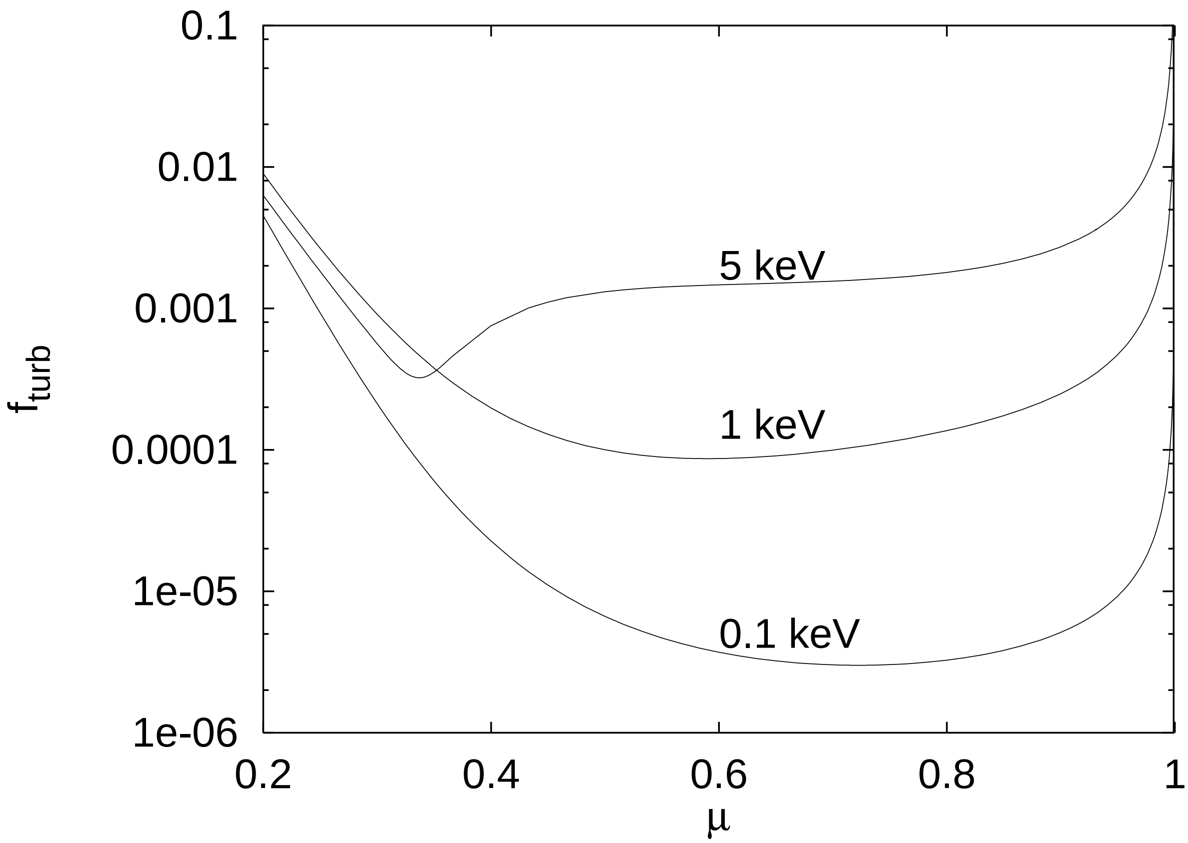

Fig. 2.— Same as Figure 1 except for the different values of temperature and for $\alpha = 0.1$, $\beta = \beta_{th}$, $q = 5/3$.

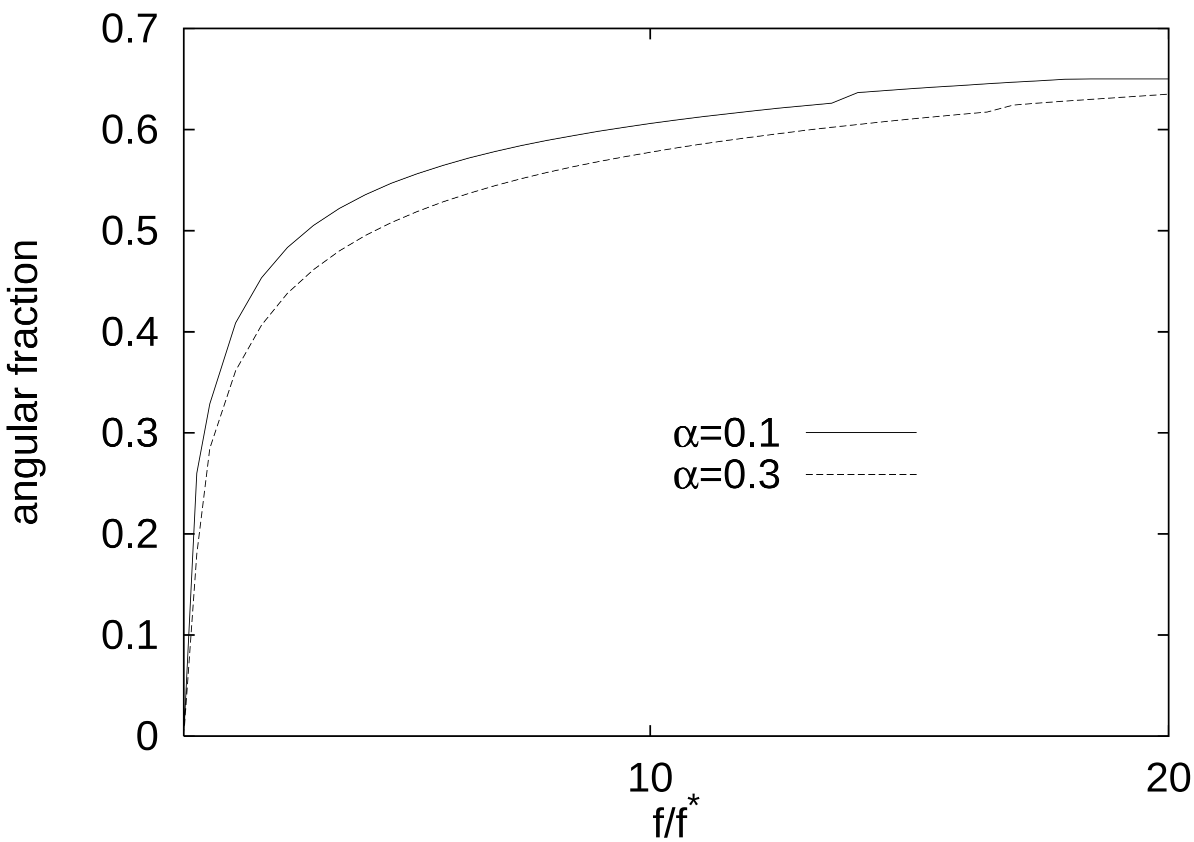

Fig. 3.— The fraction of pitch angle range of isotropicly distributed electrons with $\beta/\beta_{th} = 1$ involved in the acceleration process as a function of the ratio of the turbulent level f to the minimum turbulent level f_* (obtained from Figures 1 and 2) in plasma with $T = 1$ keV.

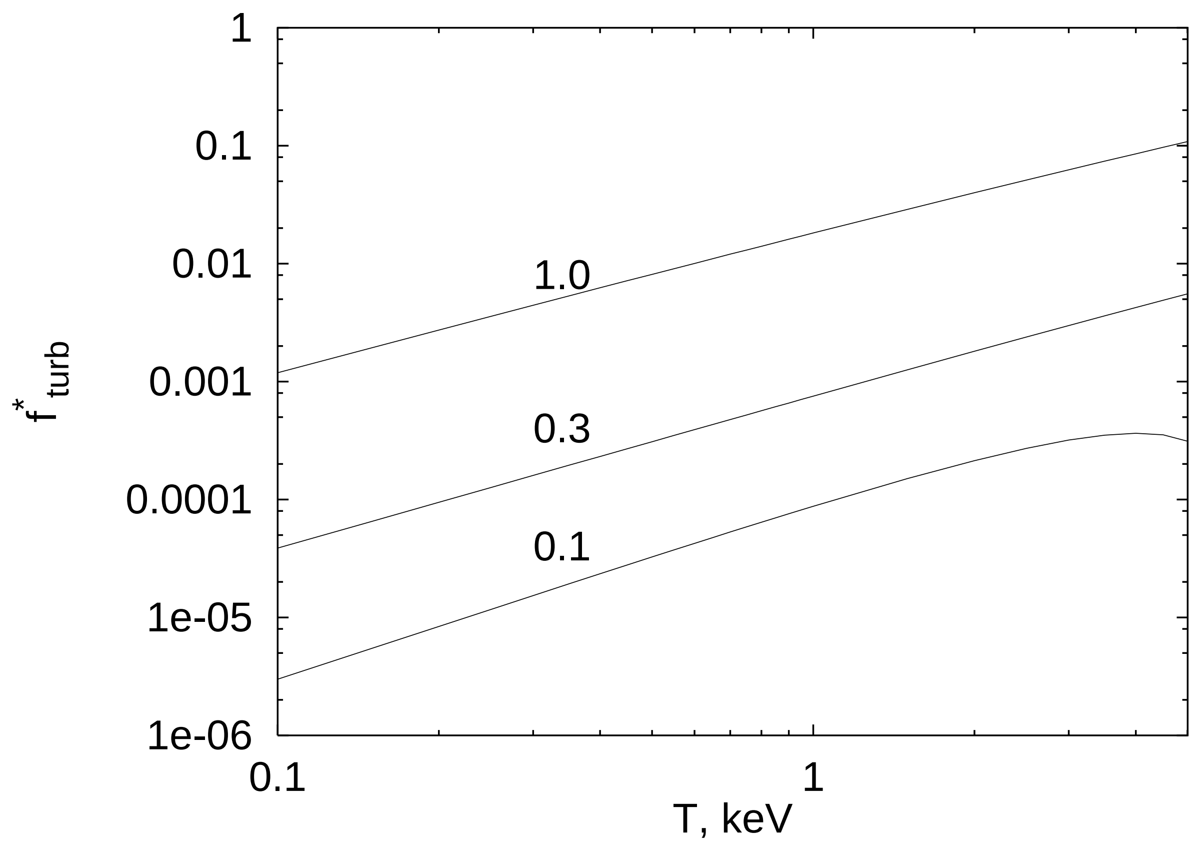

Fig. 4.— The dependence of the minimum of turbulence fraction f^*_{turb} on plasma temperature for electrons with $\beta = \beta_{th}$ in plasma with $q = 5/3$ and three different values of the parameter α.

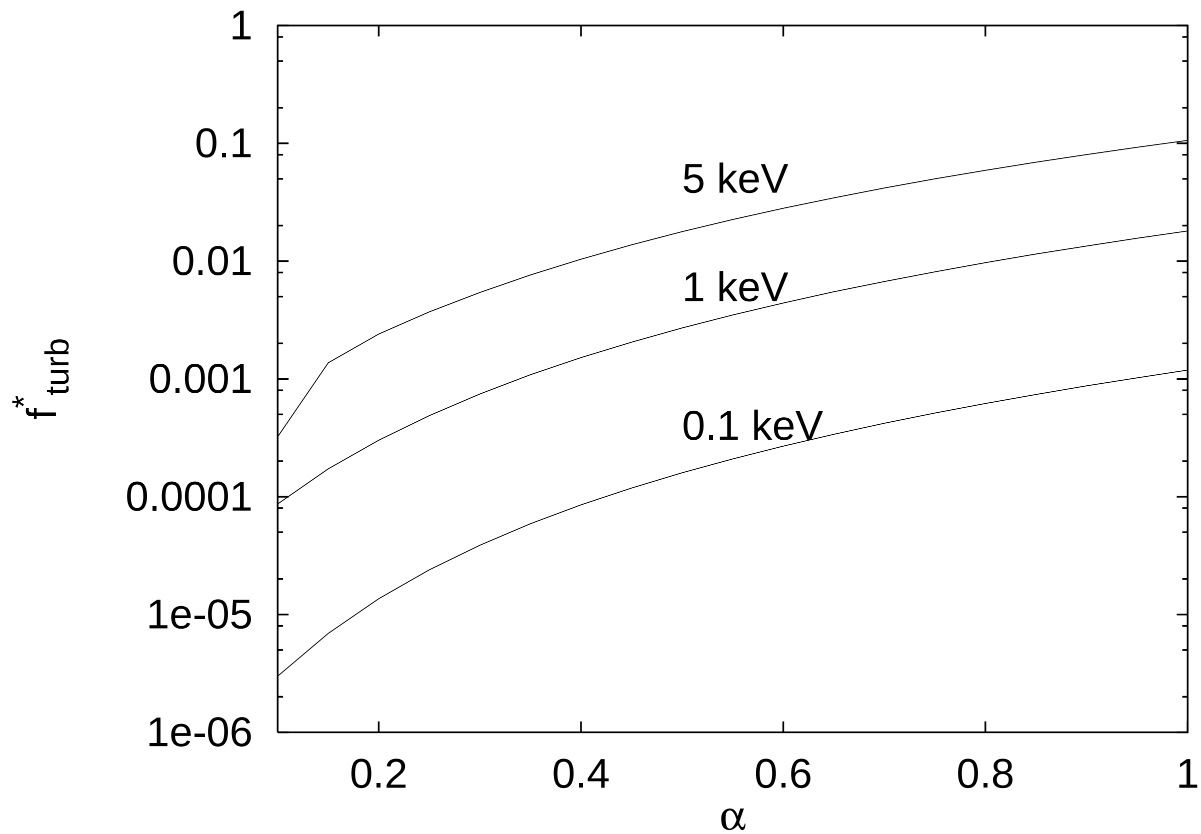

Fig. 5.— The dependence of the minimum of turbulence fraction f^*_{turb} on plasma parameter α for $q = 5/3$, $\beta = \beta_{th}$ and three different temperatures.

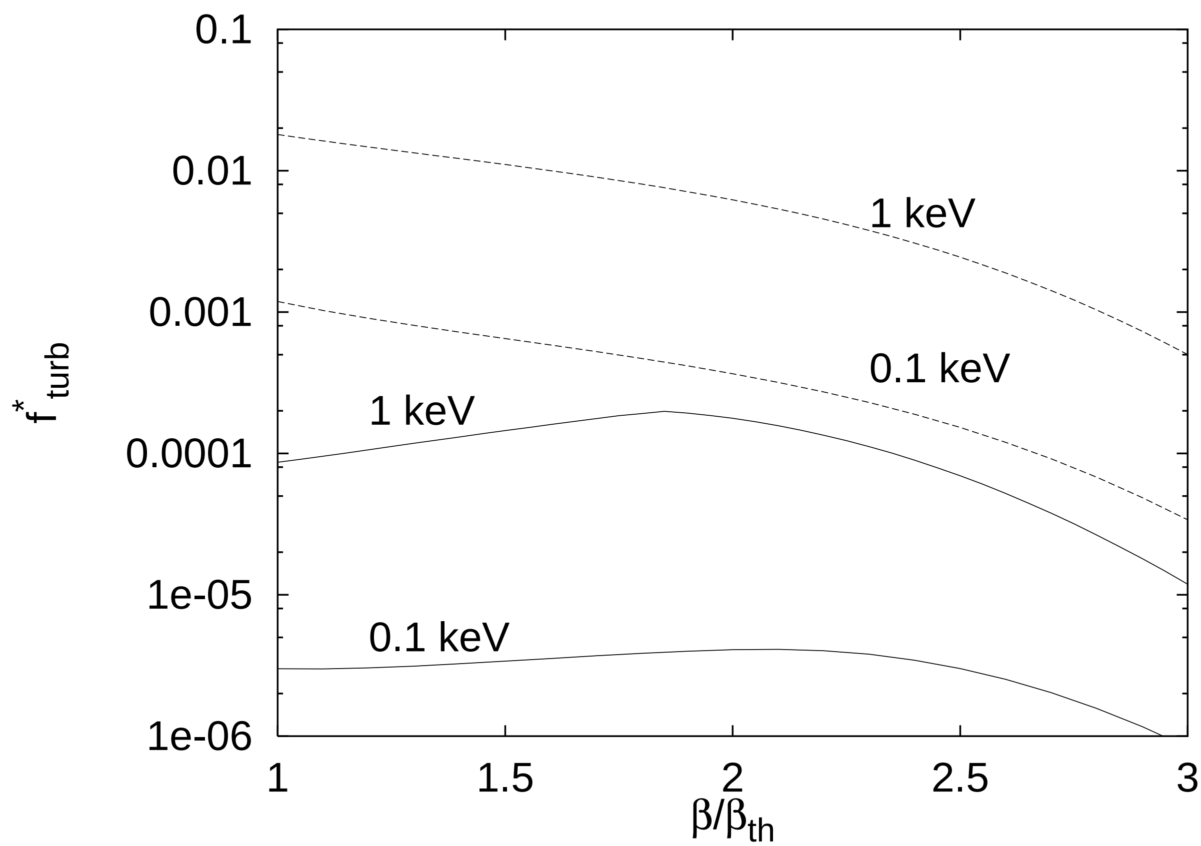

Fig. 6.— The dependence of the minimum of turbulence fraction f^*_{turb} on electron velocity (in units of thermal velocity) for $q = 5/3$ and different values of plasma parameter and temperature. Solid lines are for $\alpha = 0.1$ and dashed lines are for $\alpha = 1$.

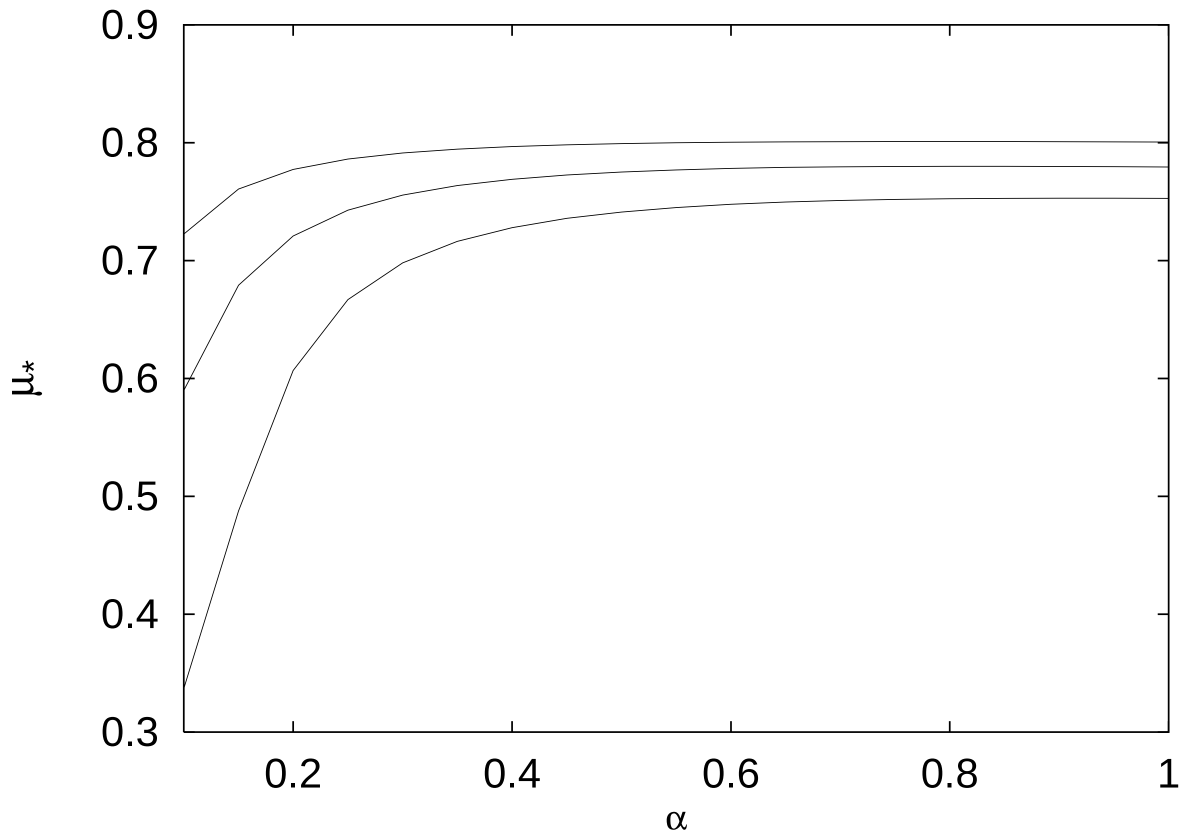

Fig. 7.— The dependence on the plasma parameter α of the critical pitch angle, at which the turbulent fraction f_{turb} is minimum, for $q = 5/3$ and three different temperatures, 0.1, 1 and 5 keV, from top to bottom.

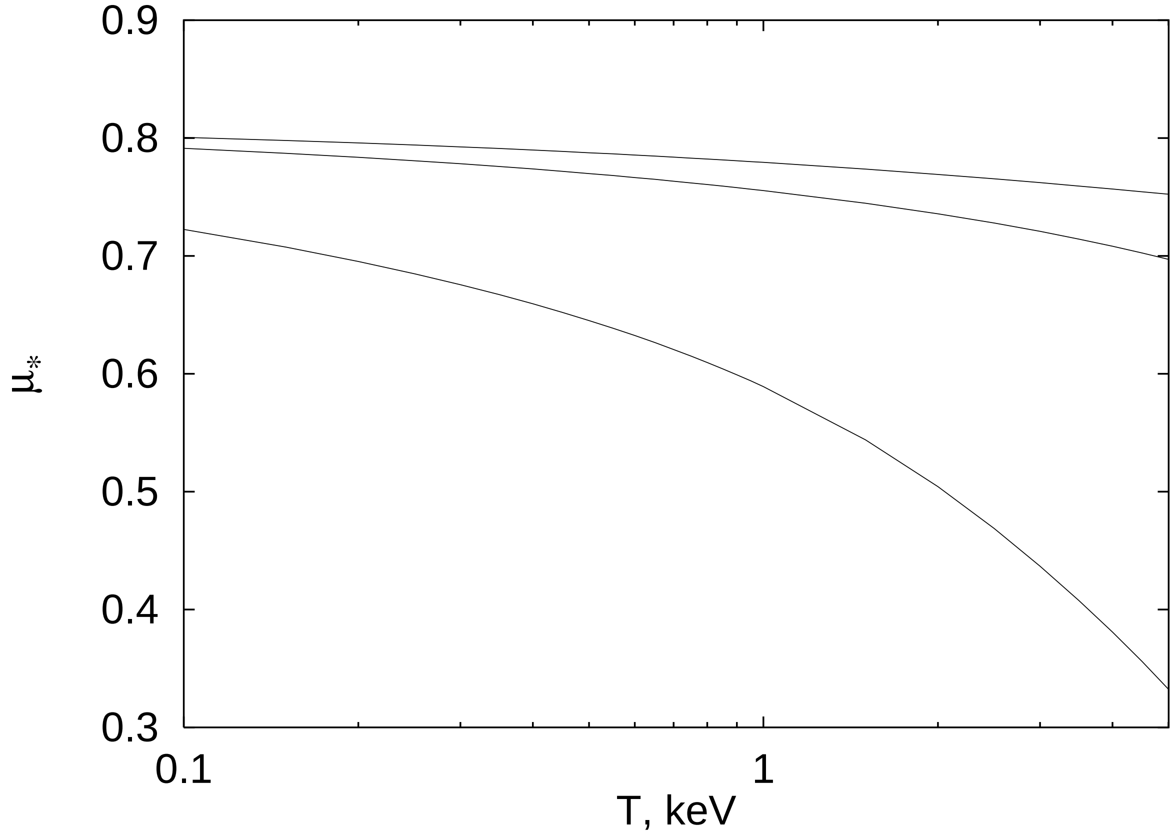

Fig. 8.— Same as Figure 7 but μ_* versus plasma temperature for $\alpha = 1$, 0.3, 0.1 from top to bottom.

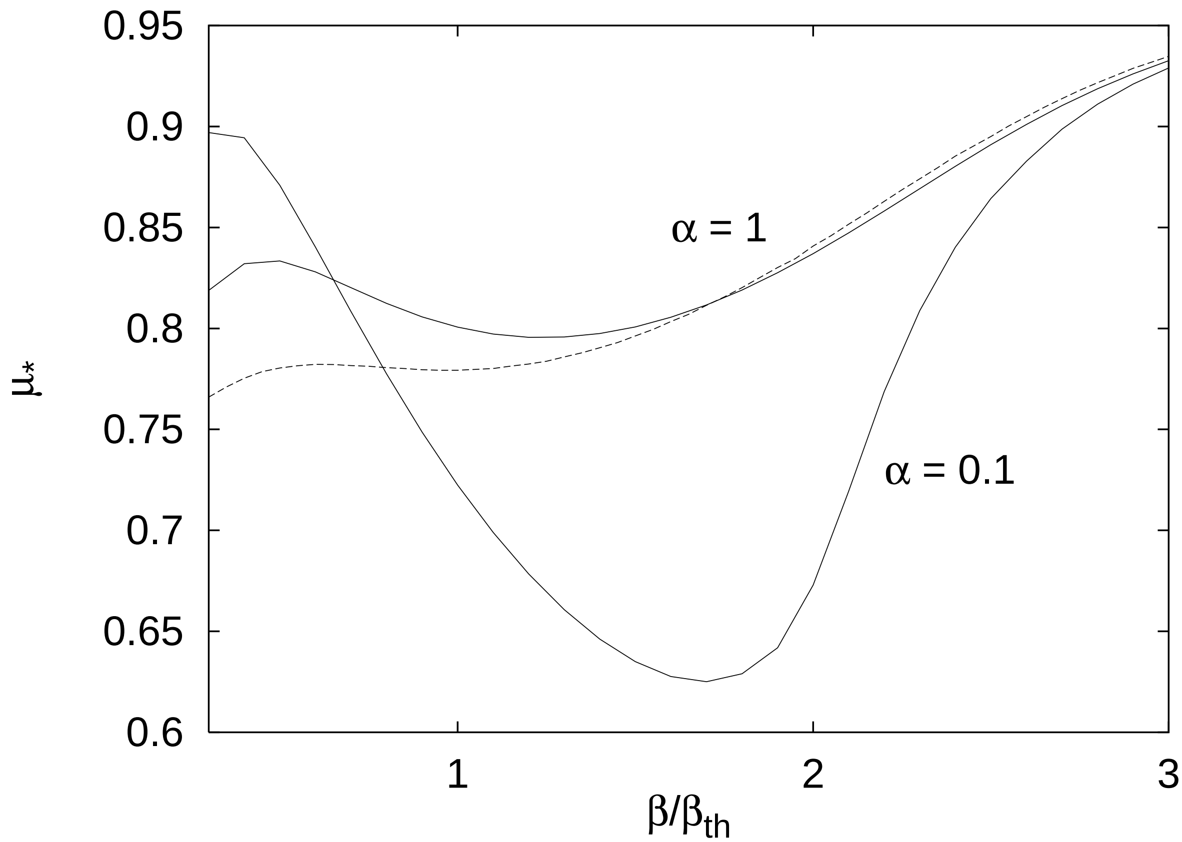

Fig. 9.— Same as Figure 7 but μ_* versus electron velocity for indicated values of α and $T = 0.1$ keV (solid lines), $T = 1$ keV (dashed line).

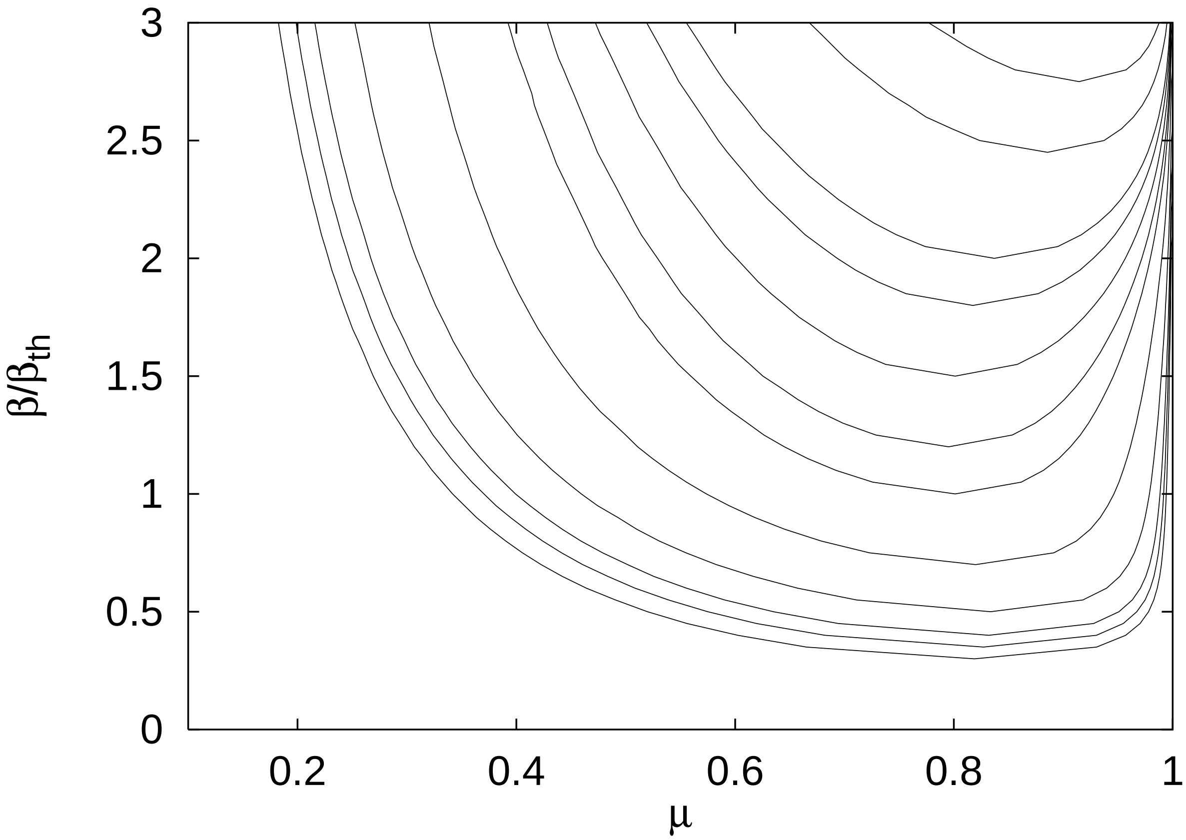

Fig. 10.— The dependence of the minimum of turbulent ratio f_{turb} on electron's velocity and pitch angle cosine in plasma with $\alpha = 1$ and $T = 0.1$ keV. The turbulent energy fraction decreases from bottom to top of the plot with values of f_{turb}: 0.016, 0.012, 0.008, 0.005, 0.002, 0.001, 0.00085, 0.0006, 0.0004, 0.0003, 0.00016, 0.00007, respectively.

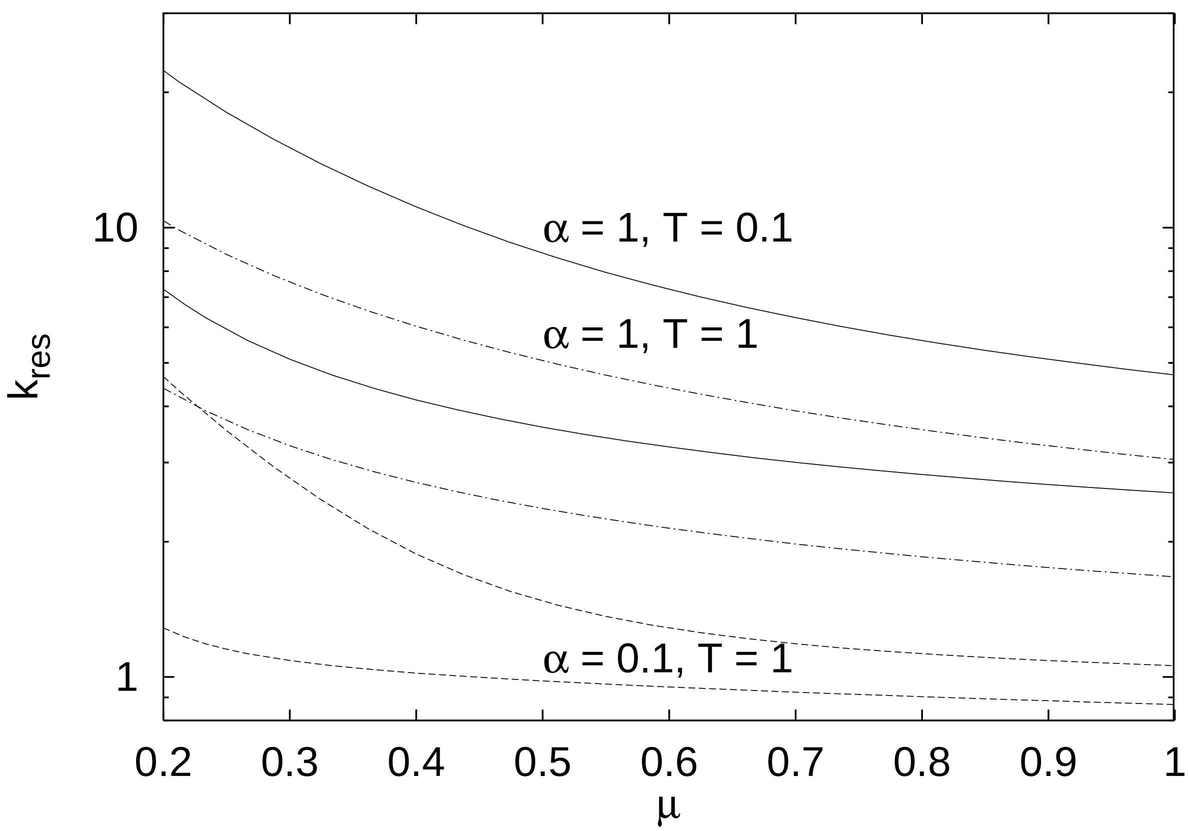

Fig. 11.— The dependence of the resonant wavevector on cosine of the electron's pitch angle. Solid lines are for a plasma with $\alpha = 1$ and $T = 0.1$ keV, dashed lines for $\alpha = 0.1$ and $T = 1$ keV, and dashed-dotted lines for $\alpha = 1$ and $T = 1$ keV. In each case the upper curve is for electrons with $\beta = \beta_{th}$ and lower curve for $\beta = 3\beta_{th}$.

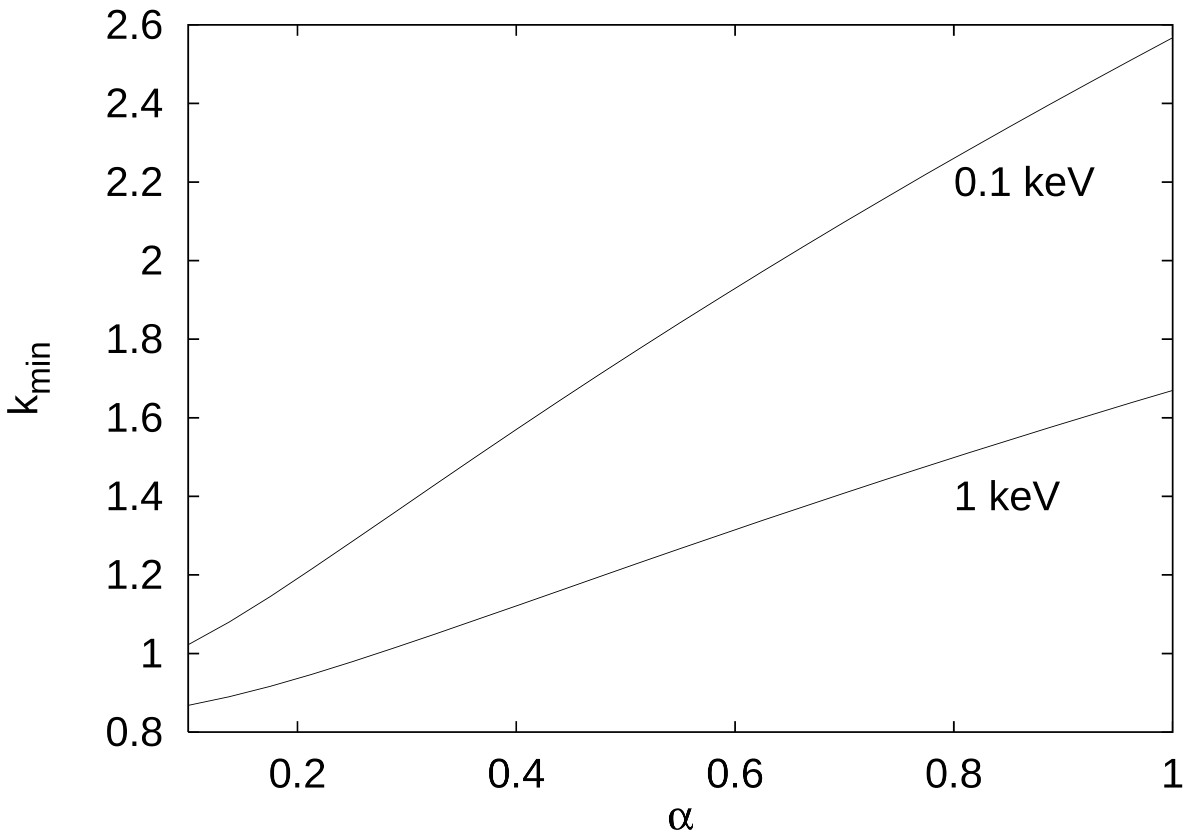

Fig. 12.— The dependence on plasma parameter α of the minimum resonant wavevector of plasma waves which interact with electrons of velocities up to $3\beta_{th}$ for two different temperatures.

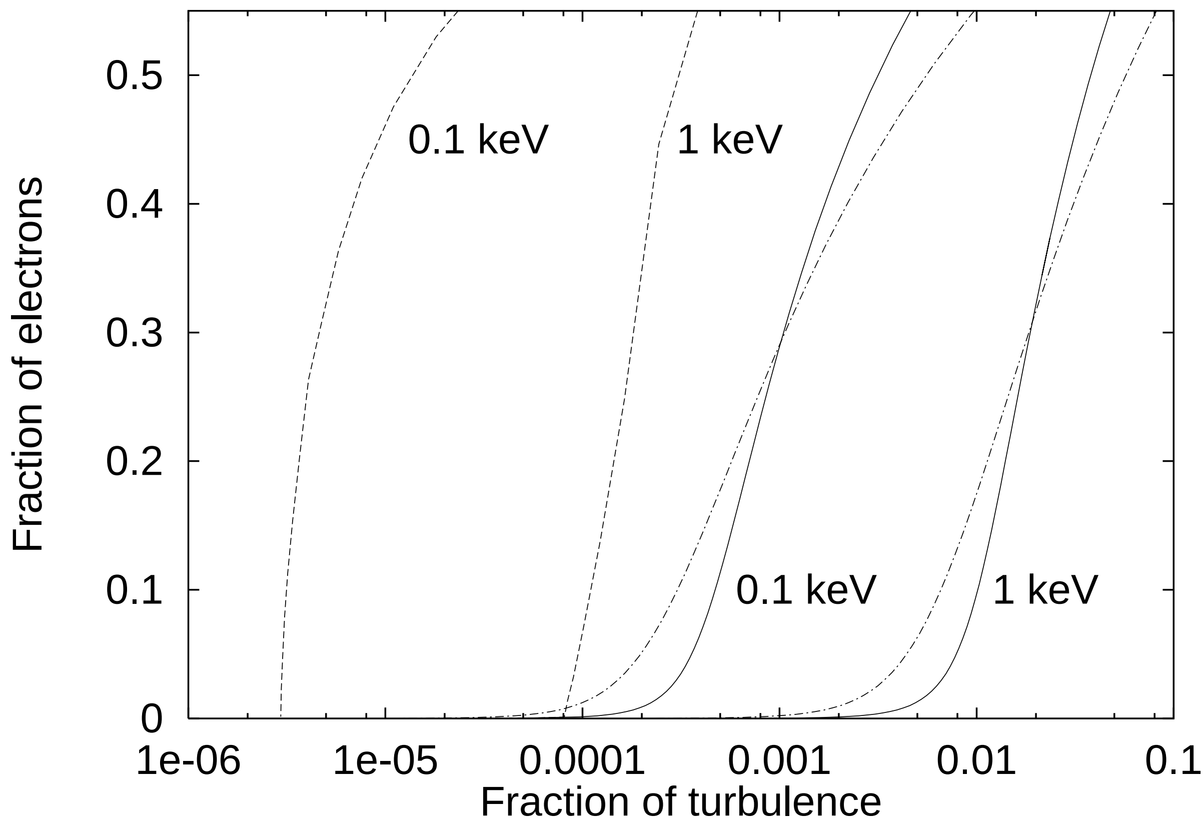

Fig. 13.— The fraction of isotropicly distributed thermal electrons involved in the acceleration process as a function of the total fraction of turbulence level $f_{turb^{tot}}$, assuming a power law up to k_{min}^{th} taken from Figure 12, for two temperatures, 0.1 and 1 keV. Dashed lines are for $\alpha = 0.1$, $q = 5/3$, solid lines for $\alpha = 1$, $q = 5/3$, dashed-dotted lines for $\alpha = 1$, $q = 3$.